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Fourth Semester B.E. Degree Examination, Dec.2018/Jan.2019
Advanced Mathematics – II

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

1. a. Prove that the angle between two lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2) is $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$ (07 Marks)
 b. Find the value of K if the angle between the lines with direction ratios $-2, 1, -1$ and $1, -K, -1$ is $\frac{2\pi}{3}$. (07 Marks)
 c. Find the projection of the line segment AB on CD where $A = (3, 4, 5)$, $B = (4, 6, 3)$, $C = (-1, 2, 4)$, $D = (1, 0, 5)$ (06 Marks)
2. a. Derive the equation of the plane in the intercept form $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (07 Marks)
 b. Find the image of the point $(2, -1, 3)$ in the plane $2x + 4y + z - 24 = 0$. (07 Marks)
 c. Find the equation of the plane containing the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$ and is perpendicular to the line $x - 2y + 3z = 4$. (06 Marks)
3. a. Show that the position vectors of the vertices of a triangle $2i - j + k$, $i - 3j - 5k$ and $3i - 4j - 4k$ form a right angled triangle. (07 Marks)
 b. Find the cosine and sine of the angle between the vectors $2i - j + 3k$ and $i - 2j + 2k$. (07 Marks)
 c. Find the value of λ such that the vectors $\vec{a} = \lambda i - 5j - 2k$, $\vec{b} = -7i + 14j - 3k$ and $\vec{c} = 11i + 4j + k$ are coplanar. (06 Marks)
4. a. A particle moves along a curve $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$. Determine its velocity and acceleration and also the magnitude of velocity and acceleration at $t = 2$. (07 Marks)
 b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. (07 Marks)
 c. Find the directional derivative of the function $\phi = xyz$ along the direction of the normal to the surface $xy^2 + yz^2 + zx^2 = 3$ at the point $(1, 1, 1)$ (06 Marks)
5. a. If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$. (07 Marks)
 b. Show that $\text{curl}(\text{grad}\phi) = 0$. (06 Marks)
 c. Show that $\vec{F} = \frac{x\vec{i} + y\vec{j}}{x^2 + y^2}$ is both solenoidal and irrotational. (07 Marks)

- 6 a. Find the Laplace transform of t^n , where n is a positive integer. (05 Marks)
 b. Find $L(\sin 5t \cos 2t)$. (05 Marks)
 c. Find $L(t \cos at)$. (05 Marks)
 d. Find $L\left(\frac{\cos at - \cos bt}{t}\right)$. (05 Marks)
- 7 a. Find $L^{-1}\left[\frac{s+5}{s^2-6s+13}\right]$. (07 Marks)
 b. Find $L^{-1}\left[\frac{1}{s(s+1)(s+2)(s+3)}\right]$. (07 Marks)
 c. Find $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$. (06 Marks)
- 8 a. Using Laplace transform solve $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}$, $y(0) = 0 = y'(0)$ (10 Marks)
 b. Using Laplace transform solve $\frac{dx}{dt} + y = \sin t$, $\frac{dy}{dt} + x = \cos t$ given $x(0) = 1, y(0) = 0$ (10 Marks)
